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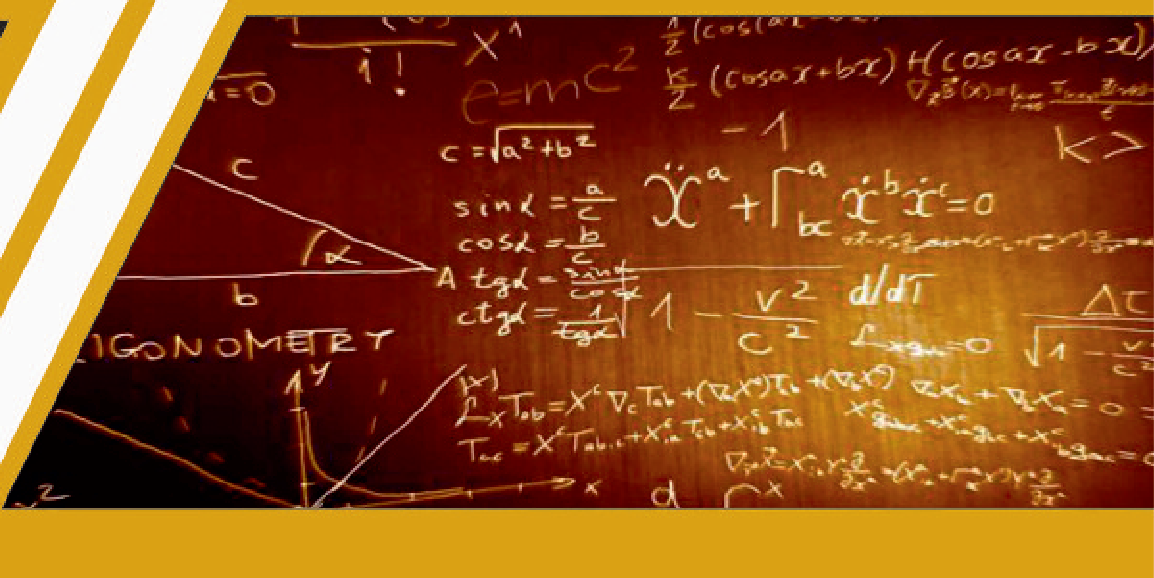

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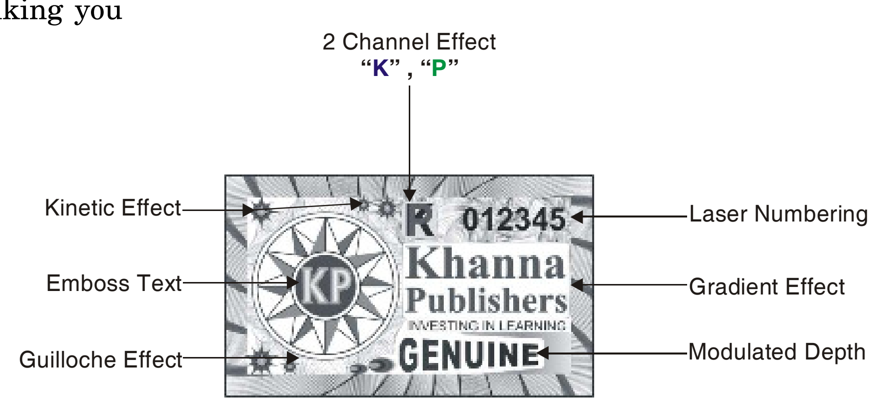

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CSIR MATHEMATICAL SCIENCES

JUNE - 2011 **FULLY SOLVED**

PART-B

21. Let $S = \{A : A = [a_{ij}]_{5\times 5}, a_{ij} = 0$ or $1 \forall i, j, \sum_j a_{ij} = 1 \forall i$ and $\sum_j a_{ij} = 1 \forall j\}$. Then the number of elements in S is

(a) 5^2 (b) 5^5

(c) $5!$ (d) 55

22. The number of 4 digit numbers with no two digits common is

(a) 4536 (b) 3024

(c) 5040 (d) 4823

23. Let D be a non-zero n x n real matrix with $n \geq 2$. Which of the following implications is valid?

(a) det $(D) = 0$ implies rank $(D) = 0$

(b) det $(D) = 1$ implies rank $(D) \neq 1$

(c) rank (D) = 1 implies det $(D) \neq 0$

(d) rank $(D) = n$ implies det $(D) \neq 1$

24. Let $f_n(x) = x^{1/n}$ for $x \in [0, 1]$. Then

(a) $\lim_{n\to\infty} f_n(x)$ exists for all $x \in [0, 1]$.

(b) $\lim_{n\to\infty} f_n(x)$ defines a continuous function on [0,1]

(c) $\{f_n\}$ converges uniformly on [0, 1]

(d) $\lim_{n\to\infty} f_n(x) = 0$ for all $x \in [0, 1]$.

25. Let $A = \{x^2 : 0 < x < 1\}$ and $B = \{x^3 : 1 < x < 2\}$. Which of the following statements is true?

(a) There is a one to one, onto function from A to B.

(b) There is no one to one, onto function from A to B taking rationals to rationals.

(c) There is no one to one function from A to B which is onto.

(d) There is no onto function from A to B which is one to one.

26. Let ξ be a primitive fifth root of unity. Define

$$A = \begin{pmatrix} \xi^{-2} & 0 & 0 & 0 & 0 \\ 0 & \xi^{-1} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \xi & 0 \\ 0 & 0 & 0 & 0 & \xi^2 \end{pmatrix}$$

For a vector $v = (v_1, v_2, v_3, v_4, v_5) \in \mathbb{R}^5$, define $|V|_A = \sqrt{|VAV^T|}$ where V^T is transpose of V. If $w = (1,-1, 1, 1, -1,)$ then $|W|_A$ equals

(a) 0 (b) 1

(c) -1 (d) 2

27. The number of elements in the set $\{m : 1 \leq m \leq 1000$, m and 1000 are relatively prime $\}$ is

(a) 100 (b) 250

(c) 300 (d) 400

28. The unit digit of 2^{100} is

(a) 2 (b) 4

(c) 6 (d) 8

29. The dimension of the vector space of all symmetric matrices of order $n \times n$ $(n \geq 2)$ with real entries and trace equal to zero is

(a) $\{(n^2 - n)/2\} - 1$ (b) $\{(n^2 + n)/2\} - 1$

(c) $\{(n^2 - 2n)/2\} - 1$ (d) $\{(n^2 + 2n)/2\} - 1$

30. Let $I = \{1\} \cup \{2\} \subset \mathbb{R}$. For $x \in \mathbb{R}$, let $\varphi(x) =$ dist $(x, I) = \inf\{|x-y| : y \in I\}$ Then

(a) φ is discontinuous somewhere on $\mathbb{R}$.

(b) φ is continuous on $\mathbb{R}$ but not differentiable only at $x = 1$.

(c) φ is continuous on $\mathbb{R}$ but not differentiable only at $x = 1$ and 2

(d) φ is continuous on $\mathbb{R}$ but not differentiable only at $x = 1$, 3/2 and 2.

31. The set $\left\{\frac{1}{n}\sin\frac{1}{n} : n \in \mathbb{N}\right\}$ has

(a) one limit point and it is 0

(b) one limit point and it is 1

(c) one limit point and it is -1

(d) three limit points and these are -1, 0 and 1

32. Using the fact that $\sum_{1}^{\infty}\frac{(-1)^{n+1}}{n} = \log 2$, $\sum_{1}^{\infty}\frac{(-1)^n}{n(n+1)}$ equals

(a) 1 - 2 log2 (b) 1 + log2

(c) $(\log 2)^2$ (d) $-(\log 2)^2$

33. Let $f : \mathbb{C} \to \mathbb{C}$ be a complex valued function given by $f(z) = u(x,y) + iv(x, y)$. Suppose that $v(x, y) = 3xy^2$. Then

(a) f cannot be holomorphic on $\mathbb{C}$ for any choices of u.

(b) f is holomorphic on $\mathbb{C}$ for a suitable choice of u.

(c) f is holomorphic on $\mathbb{C}$ for all choices of u.

(d) v is not differentiable as a function of x and y.

34. For $V = (V_1, V_2) \in \mathbb{R}^2$ and $W = (W_1, W_2) \in \mathbb{R}^2$, consider the determinant map det: $\mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ defined by det: $(V, W) = V_1W_2 - V_2W_1$. Then the derivative of the determinant map at $(V, W) \in \mathbb{R}^2 \times \mathbb{R}^2$ evaluated on $(H, K) \in \mathbb{R}^2 \times \mathbb{R}^2$ is

(a) det (H, W) + det (V, K)

(b) det (H, K)

(c) det (H, V) + det (W, K)

(d) det (V, H) + det (K, W)

35. Let W be the vector space of all real polynomials of degree at most 3. Define $T : W \to W$ by $(Tp)(x) = p'(x)$ where p' is the derivative of p. The matrix of T in the basis $\{1, x, x^2, x^3\}$, considered as column vectors, is given by

(a) $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

36. The degree of the extension $\mathbb{Q}\left(\sqrt{2} + \sqrt[3]{2}\right)$ over the field $\mathbb{Q}\left(\sqrt{2}\right)$ is

(a) 1 (b) 2

(c) 3 (d) 6

37. The power series $\sum_{0}^{\infty} 2^{-n} z^{2n}$ converges if

(a) $|z| \leq 2$ (b) $|z| < 2$

(c) $|z| \leq \sqrt{2}$ (d) $|z| < \sqrt{2}$

38. Consider a group G. Let $Z(G)$ be its centre, i.e., $Z(G) = \{g \in G : gh = hg$ for all $h \in G\}$. For $n \in \mathbb{N}$, the set of positive integers, define $J_n = \{g_1, ..., g_n\} \in Z(G) \times ... \times Z(G) : g_1 ... g_n = e\}$. As a subset of the direct product group $G \times .... \times G$ (n times direct product of the group G), J_n is

(a) not necessarily a subgroup

(b) a subgroup but not necessarily a normal sub group.

(c) a normal subgroup

(d) isomorphic to the direct product $Z(G) \times ... \times Z(G)$ ((n - 1) times).

39. Let I_1 be the ideal generated by $x^4 + 3x^2 + 2$ and I_2 be the ideal generated by $x^3 + 1$ in $\mathbb{Q}[x]$.
If $F_1 = \mathbb{Q}[x]/I_1$ and $F_2 = \mathbb{Q}[x]/I_2$, then
(a) F_1 and F_2 are fields.
(b) F_1 is a field, but F_2 is not a field.
(c) F_1 is not a field while F_2 is a field.
(d) neither F_1 nor F_2 is a field.

40. Let G be a group of order 77. Then the center of G is isomorphic to

(a) $\mathbb{Z}_{(1)}$ (b) $\mathbb{Z}_{(7)}$

(c) $\mathbb{Z}_{(11)}$ (d) $\mathbb{Z}_{(77)}$

41. Let P be a polynomial of degree N, with $N \geq 2$. Then the initial value problem $u'(t) = P(u(t))$, $u(0) = 1$ has always

(a) a unique solution in $\mathbb{R}$

(b) N number of distinct solution in $\mathbb{R}$

(c) no solution in any interval containing 0 for some P.

(d) a unique solution in an interval containing 0.

42. Consider the ODE

$u''(t) + P(t)u'(t) + Q(t)\,u(t) = R(t),\ t \in [0, 1]$

There exist continuous functions P, Q and R defined on [0, 1] and two solutions u_1 and u_2 of this ODE such that the Wronskian W of u_1 and u_2 is

(a) $W(t) = 2t - 1, \quad 0 \leq t \leq 1$

(b) $W(t) = \sin 2\pi t, \quad 0 \leq t \leq 1$

(c) $W(t) = \cos 2\pi t, \quad 0 \leq t \leq 1$

(d) $W(t) = 1, \quad 0 \leq t \leq 1$

43. The number of characteristic curves of the PDE

$(x^2 + 2y)\,u_{xx} + (y^3 - y + x)\,u_{yy} + x^2(y-1)u_{xy} - 3u_x + u = 0$

passing through the point $x = 1, y = 1$ is

(a) 0 (b) 1

(c) 2 (d) 3

44. A general solution of the second order Equation $4u_{xx} - u_{yy} = 0$ is of the form $u(x,y) =$
(a) $f(x) + g(y)$
(b) $f(x + 2y) + g(x - 2y)$
(c) $f(x + 4y) + g(x - 4y)$
(d) $f(4x + y) + g(4x - y)$, where f and g are twice differentiable functions.

45. Consider the function $f(x) = e^{-x}$ and its Taylor approximation $g(x)$ of degree 3. For $x = \frac{1}{3}$, $g(x)$ is

(a) positive and less than 1

(b) negative and less than -2

(c) positive and greater than 1

(d) less than 1 but greater than 0.75

46. The variational problem of extremizing the functional $I(y(x)) = \int_0^{2\pi} \left[\left(\frac{d}{dx} y \right)^2 - y^2 \right] dx;$

$y(0) = 1, y(2\pi) = 1$ has

(a) a unique solution

(b) exactly two solutions

(c) an infinite number of solutions

(d) no solution

47. For the Volterra type linear integral equation

$$\phi(x) = x + 2\int_0^x e^{x-\xi} \phi(\xi)\, d\xi$$

the resolvent kernel $R(x, \xi; 2)$ of the kernel $e^{x-\xi}$ is

(a) $(x-\xi)^2 e^{2(x-\xi)}$ (b) $(x-\xi)e^{x-\xi}$

(c) $e^{3(x-\xi)}$ (d) $e^{(x-\xi)}$

48. Which of the following is/are correct

(a) A free particle in $\mathbb{R}^3$ can have infinite degrees of freedom

(b) The number of degree of freedom of N particles is greater than $3N$

(c) A system of N particles with k constants has $3N + k$ degrees of freedom

(d) A system consisting of three point masses connected by three rigid massless rods has six degrees of freedom.

49. A system of 5 identical units consists of two parts A and B which are connected in series. Part A has 2 units connected in parallel and part B has 3 units connected in parallel. All the 5 units function independently with probability of failure $\frac{1}{2}$. Then the reliability of the system is

(a) $\frac{31}{32}$ (b) $\frac{11}{32}$

(c) $\frac{1}{32}$ (d) $\frac{21}{32}$

50. Suppose $X_1, X_2, ...$ is an i.i.d. sequence of random variables with common variance $\sigma^2 > 0$. Let $Y_n = \frac{1}{n}\sum_{i=1}^{n} X_{2i-1}$ and $Z_n = \frac{1}{n}\sum_{i=1}^{n} X_{2i}$. Then the asymptotic distribution (as $n \to \infty$) of $\sqrt{n}\left(Y_n - Z_n\right)$ is

(a) $N(0, 1)$ (b) $N(0, \sigma^2)$

(c) $N(0, 2\sigma^2)$ (d) degenerate at 0

51. Consider an aperiodic Markov chain with state space S and with stationary transition probability matrix $P = ((p_{ij}))$, $i,j \in S$. Let the n-step transition probability matrix be denoted by $P^n = ((p_{ij}^n))$, i, $j \in S$. Then which of the following statements is true?

(a) $\lim_{n\to\infty} p_{ii}^n = 0$ only if i is transient.

(b) $\lim_{n\to\infty} p_{ii}^n > 0$ if and only if i is recurrent.

(c) $\lim_{n\to\infty} p_{ij}^n = \lim_{n\to\infty} p_{jj}^n$ if i and j are in the same communicating class.

(d) $\lim_{n\to\infty} p_{ij}^n = \lim_{n\to\infty} p_{ii}^n$ if i and j are in the same communicating class.

52. Suppose X is a random variable with $E(X)=\text{Var}(X)$. Then the distribution of X

(a) is necessarily poisson

(b) is necessarily exponential

(c) is necessarily normal

(d) cannot be identified form the given data.

53. Let $x = 10$ be an observation on the hypergeometric random variable X, namely

$$P(X = x) = \frac{\binom{m}{x}\binom{N-m}{n-x}}{\binom{N}{n}}, x = 0,1,...,$$

$\min\{m,n\}$ and $n - x \leq N - m$, where $m = 40$, $n = 30$ and N is an unknown parameter. The maximum likelihood estimator of N is

(a) 75 (b) 120

(c) 60 (d) not unique

54. Let $X_1, X_2, ..., X_n$, $n \geq 2$, be i.i.d. observations from $N(0, \sigma^2)$ distribution, where $0 < \sigma^2 < \infty$ is an unknown parameter. Then the uniformly minimum variance unbiased estimate for σ^2 is

(a) $\frac{1}{n}\sum_{i=1}^{n} X_i^2$ (b) $\frac{1}{n-1}\sum_{i=1}^{n} X_i^2$

(c) $\frac{1}{n}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$ (d) $\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$

55. Suppose that we have i.i.d. observations (X_1, Y_1), (X_2, Y_2),....,(X_n, Y_n), $n \geq 3$, where X_i and Y_i are independent normal random variables. Consider τ = the sample Kendall's rank correlation coefficient computed from this data. Then which of the following is correct?

(a) $P(\tau > 0) > \frac{1}{2}$ (b) $P(\tau < 0) > \frac{1}{2}$

(c) $E(\tau) = 0$ (d) $E(\tau) \neq 0$

56. The reaction time to a stimulus X (in seconds) is distributed normally in group 1 with mean 2 and variance 8;group 2 with mean 4 and variance 1. The two groups appear in equal proportions. x is an observable value of X. The best discriminant function (in the sense of minimizing misclassification probabilities) is to classify intogroup

(a) 2 if $x > 3$; otherwise in group 1

(b) 1 if $x > 3$; otherwise in group 2

(c) 2 if $0 \leq x \leq \frac{8}{3}$; otherwise in group 1

(d) 1 if $0 \leq x \leq \frac{8}{3}$; otherwise in group 2

57. Batteries for torch lights are packed in boxes of 10 and a lot contains 10 boxes. A quality inspector randomly chooses a box and then checks two batteries selected randomly without replacement from that box. The lot will be rejected, if any one of the two chosen batteries turns out to be defective. Suppose that 9 of the10 boxes in the lot contain no defective batteries and only one box contains 2 defective ones. What is the probability that the lot will not be passed by the Inspector?

(a) $\frac{197}{4950}$ (b) $\frac{98}{2475}$

(c) $\frac{8}{225}$ (d) $\frac{17}{450}$

58. To examine whether two different skin creams, A and B, have different effect on the human body n randomly chosen persons were enrolled in a clinical trial. Then cream A was applied to one of the randomly chosen arms of each person, cream B to the other. What kind of a design is this?

(a) Completely Randomized Design

(b) Balanced Incomplete Block Design

(c) Randomized Block Design

(d) Latin Square Design

59. Cosider the LP problem maximize $x_1 + x_2$ subject to $x_1 - 2x_2 \le 10, x_2 - 2x_1 \le 10,\ x_1, x_2 \ge 0$. Then

(a) The LP problem admits an optimal solution

(b) The LP problem is unbounded

(c) The LP problem admits no feasible solution

(d) The LP problem admits a unique feasible solution

60. Let $X(t)$ be the number of customers in an $M/M/1$ queueing system with arrival rate 3 and service rate 6. Which of the following is true?

(a) $\lim_{t\to\infty} P\left(X(t) \ge 5\right) = 0$

(b) $\lim_{t\to\infty} P\left(X(t) \ge 5\right) = \frac{1}{32}$

(c) $\lim_{t\to\infty} P\left(X(t) \ge 5\right) = \frac{31}{32}$

(d) $\lim_{t\to\infty} P\left(X(t) \ge 5\right) = 1$

PART-C

UNIT - I

61. Consider the function $f(x) = |\cos x| + |\sin(2-x)|$. At which of the following points is f not differentiable?

(a) $\left\{(2n+1)\frac{\pi}{2} : n \in \mathbb{Z}\right\}$

(b) $\{n\pi : n \in \mathbb{Z}\}$

(c) $\{n\pi + 2 : n \in \mathbb{Z}\}$

(d) $\left\{\frac{n\pi}{2} : n \in \mathbb{Z}\right\}$

62. Which of the following subsets of $\mathbb{R}^2$ are convex?

(a) $\{(x, y) : |x| \le 5\ |y| \le 10\}$

(b) $\{(x, y) : x^2 + y^2 = 1\}$

(c) $\{(x, y) : y \ge x^2\}$

(d) $\{(x, y) : y \le x^2\}$

63. Which of the following is/are metrics on $\mathbb{R}$?

(a) $d(x, y) = \min(x, y)$

(b) $d(x, y) = |x - y|$

(c) $d(x, y) = |x^2 - y^2|$

(d) $d(x, y) = |x^3 - y^3|$

64. Let X denote the two-point set {0, 1} and write X_j={0, 1} for every j = 1,2,3,... Let $y = \prod_{j=1}^{\infty} X_j$. Which of the following is/are true?

(a) Y is a countable set

(b) Card Y = card [0, 1]

(c) $\bigcup_{n=1}^{\infty}\left(\prod_{j=1}^{n} X_j\right)$ is uncountable

(d) Y is uncountable

65. Which of the following is/are correct?

(a) $n\log\left(1+\frac{1}{n+1}\right)\to 1$ as $n\to\infty$

(b) $(n+1)\log\left(1+\frac{1}{n}\right)\to 1$ as $n\to\infty$

(c) $n^2\log\left(1+\frac{1}{n}\right)\to 1$ as $n\to\infty$

(d) $n\log\left(1+\frac{1}{n^2}\right)\to 1$ as $n\to\infty$

66. If $\{x_n\}$ and $\{y_n\}$ are sequences of real numbers, which of the following is/are true?

(a) $\limsup_n (x_n+y_n) \le \limsup_n x_n + \limsup_n y_n$

(b) $\limsup_n (x_n+y_n) \ge \limsup_n x_n + \limsup_n y_n$

(c) $\liminf_n (x_n+y_n) \le \liminf_n x_n + \liminf_n y_n$

(d) $\liminf_n (x_n+y_n) \ge \liminf_n x_n + \liminf_n y_n$

67. Let $\{f_n\}$ be a sequence of integrable functions defined on an interval $[a, b]$. Then

(a) If $f_n(x)\to 0$ a.e., then $\int_a^b f_n(x)dx \to 0$

(b) If $\int_a^b f_n(x)dx\to 0$, then $f_n(x)\to 0$ a.e.

(c) If $f_n(x)\to 0$ a.e. and each f_n is a bounded function, then $\int_a^b f_n(x)dx\to 0$,

(d) If $f_n(x)\to 0$ a.e. and the f_n 's are uniformly bounded, then $\int_a^b f_n(x)dx\to 0$

68. For $x=(x_1, x_2,....,x_d)\in\mathbb{R}^d$, and $p\ge 1$ define

$$\|x\|_p=\left(\sum_{j=1}|x_j|^p\right)^{1/p}$$ and

$\|x\|_\infty=\max\{|x_j| : j=1,2,...d\}$. Which of the following inequalities hold for all $x\in\mathbb{R}^d$?

(a) $\|x\|_1\ge\|x\|_2\ge\|x\|_\infty$ (b) $\|x\|_1\le d\|x\|_\infty$

(c) $\|x\|_1\le\sqrt{d}\|x\|_\infty$ (d) $\|x\|_1\le\sqrt{d}\|x\|_2$

69. Consider the map $f:\mathbb{R}^2\to\mathbb{R}^2$ defined by $f(x, y)=(3x-2y+x^2, 4x+5y+y^2)$. Then

(a) f is discontinuous at (0, 0)

(b) f is continuous at (0, 0) and all directional derivatives exist at (0, 0)

(c) f is differentiable at (0, 0) but the derivative $Df(0, 0)$ is not invertible.

(d) f is differentiable at (0, 0) and the derivative $Df(0, 0)$ is invertible

70. Which of the following sets are dense in $\mathbb{R}^2$ with respect to the usual topology.

(a) $\{(x, y)\in\mathbb{R}^2 : x\in\mathbb{N}\}$

(b) $\{(x, y)\in\mathbb{R}^2 : x+y \text{ is a rational number}\}$

(c) $\{(x, y)\in\mathbb{R}^2 : x+y^2=5\}$

(d) $\{(x, y)\in\mathbb{R}^2 : xy\ne 0\}$

71. Let $F=\{f:\mathbb{R}\to\mathbb{R} : |f(x)-f(y)|\le K|(x-y)|^\alpha\}$ for all $x, y\in\mathbb{R}$ and for some $\alpha>0$ and some $K>0$. Which of the following is/are true?

(a) every $f\in F$ is continuous

(b) every $f\in F$ is uniformly continuous

(c) every differentiable function f is in F.

(d) every $f\in F$ is differentiable.

72. Let $a_{ij}=a_ia_j$, $1<i, j\le b$, where $a_1,....,a_n$ are real numbers. Let $A=((a_{ij}))$ be the $n\times n$ matrix $((a_{ij}))$. Then

(a) It is possible to choose $a_1,....,a_n$ so as to make the matrix A non-singular.

(b) The matrix A is positive definite if $(a_1,....,a_n)$ is a non-zero vector

(c) The matrix A is positive semi-definite for all $(a_1,....,a_n)$.

(d) For all $(a_1,....,a_n)$, zero is an eigenvalue of A.

73. Suppose A, B are $n\times n$ positive definite matrices and I be the $n\times n$ identity matrix,then which of the following are positive definite.

(a) $A+B$ (b) ABA^*

(c) A^2+I (d) AB

74. Let T be a linear transformation on the real vector space $\mathbb{R}^n$ over $\mathbb{R}$ such that $T^2 = \lambda T$ for some $\lambda \in \mathbb{R}$. Then

(a) if $\|Tx\| = |\lambda|\|x\|$ for all $x\in\mathbb{R}^n$

(b) If $\|Tx\| = \|x\|$ for some non-zero vector $x\in\mathbb{R}^n$, then $\lambda=\pm 1$

(c) $T = \lambda I$ where I is the identity transformation on $\mathbb{R}^n$.

(d) If $\|Tx\| > \|x\|$ for a non-zero vector $x\in\mathbb{R}^n$, then T is necessarily singular.

75. Let M be the vector space of all 3 x 3 real matrices and let $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$. Which of the following are subspaces of M?

(a) $\{X\in M : XA = AX\}$

(b) $\{X\in M : X+A = A+X\}$

(c) $\{X\in M : \text{trace}(AX) = 0\}$

(d) $\{X\in M : \det(AX) = 0\}$

76. Let $W = \{p(B) : p$ is a polynomial with real coefficients$\}$, where $B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$. The dimension d of the vector space W satisfies

(a) $4\le d\le 6$ (b) $6\le d\le 9$

(c) $3\le d\le 8$ (d) $3\le d\le 4$

77. Let N be a 3×3 nonzero matrix with the property $N^3 = 0$. Which of the following is/are true?

(a) N is not similar to a diagonal matrix.

(b) N is similar to a diagonal matrix.

(c) N has one non-zero eigen vector.

(d) N has three linearly independent eigen vector.

78. Let $x, y\in\mathbb{C}^n$. Consider

$$f(x,y) = \sup_{\theta,\varphi}\left\|e^{i\theta}x - e^{i\varphi}y\right\|_2 \;,\theta,\phi\in\mathbb{R}.$$

Which of the following is/are correct?

(a) $f(x,y)\le \|x\|^2+\|y\|^2 - 2\,Re\left|\langle x,y\rangle\right|$

(b) $f(x,y)\le \|x\|^2+\|y\|^2 +2\,Re\left|\langle x,y\rangle\right|$

(c) $f(x,y)= \|x\|^2+\|y\|^2 +2\left|\langle x,y\rangle\right|$

(d) $f(x,y)\ge \|x\|^2+\|y\|^2 - 2\,Re\left|\langle x,y\rangle\right|$

UNIT - II

79. Let $D=\{z\in\mathbb{C} : |z|<1\}$ be the unit disc. Let $f : D\to\mathbb{C}$ be an analytic function satisfying $f\left(\frac{1}{n}\right)=\frac{2n}{3n+1}$ for $n\ge 1$. Then

(a) $f(0) = 2/3$

(b) f has a simple pole at $z = -3$

(c) $f(3) = 1/3$

(d) no such f exists

80. Let f be an entire function. If $\text{Re}(f)$ is bounded then

(a) $\text{Im}(f)$ is constant

(b) f is constant

(c) $f\equiv 0$

(d) f' is non zero constant

81. Let $f : D\to D$ be holomorphic with $f(0) = \frac{1}{2}$ and $f(1/2) = 0$, where $D = \{z : |z|\le 1\}$. Which of the following is correct?

(a) $|f'(0)|\le 3/4$

(b) $|f'(1/2)|\le 4/3$

(c) $|f'(0)|\le 3/4$ and $|f'(1/2)|\le 4/3$

(d) $f(z) = z, z\in D$

82. Define $H^+ = \{z\in\mathbb{C} : y>0\}$

$H^- = \{z\in\mathbb{C} : y<0\}$

$L^+ = \{z \in \mathbb{C} : x > 0\}$

$L^- = \{z \in \mathbb{C} : x < 0\}$

The function $f(z) = \dfrac{z}{3z+1}$

(a) maps H^+ onto H^+ and H^- onto H^-

(b) maps H^+ onto H^- and H^- onto H^+

(c) maps H^+ onto L^+ and H^- onto L^-

(d) maps H^+ onto L^+ and H^- onto L^+

83. At $z = 0$ the function $f(z) = \dfrac{e^z + 1}{e^z - 1}$

(a) a removable singularity

(b) a pole

(c) an essential singularity

(d) the residue of $f(z)$ at $z = 0$ is 2.

84. Let $H = \{e, (1,2)(3,4)\}$ and $K = \{e, (1,2)(3,4), (1,3)(2,4), (1,4)(2,3)\}$ be subgroups of S_4, where e denotes the identify element of S_4. Then,

(a) H and K are normal subgroups of S_4.

(b) H is normal in K and K is normal in A_4.

(c) H is normal in A_4 but not normal in S_4.

(d) K is normal in S_4 but H is not.

85. Let $<p(x)>$ denote the ideal generated by the polynomial $p(x)$ in $\mathbb{Q}[x]$. If $f(x) = x^3 + x^2 + x + 1$ and $g(x) = x^3 - x^2 + x - 1$, then

(a) $<f(x)> + <g(x)> = <x^3 + x>$

(b) $<f(x)> + <g(x)> = <f(x) . g(x)>$

(c) $<f(x)> + <g(x)> = <x^2 + 1>$

(d) $<f(x)> + <g(x)> = <x^4 - 1>$

86. Let I_1 be the ideal generated by $x^2 + 1$ and I_2 be the ideal generated by $x^3 - x^2 + x - 1$ in $\mathbb{Q}[x]$.

If $R_1 = \mathbb{Q}[x]/I_1$ and $R_2 = \mathbb{Q}[x]/I_2$. Then

(a) R_1 and R_2 are fields

(b) R_1 is a field and R_2 is not a field

(c) R_1 is an integral domain, but R_2 is not an integral domain.

(d) R_1 and R_2 are not integral domains.

87. Let $G = \mathbb{Z}_{10} \times \mathbb{Z}_{15}$. Then,

(a) G contains exactly one element of order 2

(b) G contains exactly 5 elements of order 3

(c) G contains exactly 24 elements of order 5

(d) G contains exactly 24 elements of order 10

88. The space C [0, 1] of continuous functions on [0, 1] is complete with respect to which of the following

(a) $\|f\|_\infty = \sup\{|f(x)| : x \in [0,1]\}$

(b) $\|f\|_2 = \left(\int_0^1 |f(x)|^2 \, dx\right)^{\frac{1}{2}}$

(c) $\|f\|_{\infty,½} = \|f\|_\infty + |f(1/2)|$

(d) $\|f\|_{\infty,}$ and $= \|f\|_{\infty,½}$

89. Consider the set

$$X = (-\infty, 0] \bigcup \left\{\frac{1}{n} : n \in \mathbb{N}\right\} \subseteq \mathbb{R}$$

with the subspace topology. Then

(a) 0 is an isolated point

(b) (-2, 0] is an open set

(c) 0 is a limit point of the subset $\left\{\dfrac{1}{n} : n \in \mathbb{N}\right\}$

(d) (-2, 0) is an open set

90. Consider three subsets of $\mathbb{R}^2$, namely

$A_1 = \{(x, y) : x^2 + y^2 \le 1\}$

$A_2 = \{(1, y) : y \in \mathbb{R}\}$

$A_3 = \{(0, 2)\}$.

Then, there always exists a continuous real valued function f on $\mathbb{R}^2$ such that $f(x) = a_j$ for $x \in A_j, j = 1, 2, 3$

(a) if and only if at least two of the numbers a_1, a_2, a_3 are equal

(b) if $a_1 = a_2 = a_3$

(c) for all real values of a_1, a_2, a_3

(d) if and only if $a_1 = a_2$

UNIT - III

91. The Green's function $G(x, \zeta)$, $0 \le x, \zeta \le 1$ of the boundary value problem $y'' + \lambda y = 0$, $y(0) = 0 = y(1)$ is

(a) symmetric in x and ζ

(b) continuous at $x = \zeta$

(c) $\left.\frac{\partial G(x,\zeta)}{\partial x}\right|_{x=\zeta^-} - \left.\frac{\partial G(x,\zeta)}{\partial x}\right|_{x=\zeta^+} = -1$

(d) $\left.\frac{\partial G(x,\zeta)}{\partial x}\right|_{x=\zeta^-} - \left.\frac{\partial G(x,\zeta)}{\partial x}\right|_{x=\zeta^+} = 1$

92. For the boundary value problem,

$$y'' + \lambda y = 0$$
$$y(-\pi) = y(\pi),$$
$$y'(-\pi) = y'(\pi),$$

to each eigenvalue λ, there corresponds

(a) only one eigen function

(b) two eigen functions

(c) two linearly independent eigen functions

(d) two orthogonal eigen functions

93. Let $y_1(x)$ and $y_2(x)$ form a fundamental set of solutions to the differential equation

$$\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + q(x)y = 0, \quad 0 \le x \le b,$$

where $p(x)$ and $q(x)$ are continuous in $[a, b]$ and x_0 is a point in (a, b). Then

(a) both $y_1(x)$ and $y_2(x)$ cannot have a local maximum at x_0

(b) both $y_1(x)$ and $y_2(x)$ cannot have a local minimum at x_0.

(c) $y_1(x)$ cannot have a local maximum at x_0 and $y_2(x)$ cannot have a local minimum at x_0 simultaneously.

(d) both $y_1(x)$ and $y_2(x)$ cannot vanish at x_0 simul taneously.

94. A general solution of the PDE $uu_x + yu_y = x$ is of the form

(a) $f\left(u^2 - x^2, \frac{y}{x+u}\right) = 0$, where $f : \mathbb{R}^2 \to \mathbb{R}$ is C^1 and $\nabla f \ne (0, 0)$ at every point

(b) $u^2 = g\left(\frac{y}{x+u}\right) + x^2$, $g \in C^1(\mathbb{R})$

(c) $f\left(u^2 + x^2\right) = 0$, $f \in C^1(\mathbb{R})$

(d) $f(x + y) = 0$, $f \in C^1(\mathbb{R})$

95. The PDE

$$\left.\begin{aligned} &u_{xx} + u_{yy} + \lambda u = 0, \quad 0 < x, y < 1 \\ &u(x,0) = u(x,1) = 0, \quad 0 \le x \le 1 \\ &u(0,y) = u(1,y) = 0, \quad 0 \le y \le 1 \end{aligned}\right\} \text{ has}$$

(a) a unique solution u for any $\lambda \in \mathbb{R}$.

(b) infinitely many solutions for some $\lambda \in \mathbb{R}$.

(c) a solution for countably many values of λ

(d) infinitely many solutions for all $\lambda \in \mathbb{R}$.

96. The Cauchy problem

$$\begin{aligned} u_x(x,y) + u_y(x,y) = 0 &\quad \text{for } (x,y) \in \mathbb{R}^2 \\ u(x,y) = 0 &\quad \text{for all } x \in \mathbb{R} \end{aligned} \quad \text{has}$$

(a) a unique solution

(b) a family of straight lines as characteristics

(c) solution which vanishes at (2, 1)

(d) infinitely many solutions.

97. Consider a linear system $Ax = b$ with a computed solution x_C; the error and the residue are defined, respectively by

$$e = x - x_c$$
$$r = Ax - Ax_c$$

Then

(a) a small error necessarily implies a small residue

(b) The error can be larger with relatively small residue.

(c) The error can be small with relatively large residue

(d) The error and the residue are always equal.

98. Consider the iteration function for Newton's method $g(x) = x - \frac{f(x)}{f'(x)}$ and its application to find (approximate) square root of 2, starting with $x_0 = 2$. Consider the first and the second iterates x_1 and x_2, respectively; then

(a) $1.5 \leq x_1 \leq 2$

(b) $1.5 \leq x_1 < 2$

(c) $x_1 \leq 1.5;\ x_2 \leq 1.5$

(d) $x_1 = 1.5;\ x_2 < 1$

99. In the Ritz method, seeking an extremum of the functional $I(y) = \int_{x_o}^{x_1} F\left(x, y, \frac{dy}{dx}\right) dx;$

$$y(x_0) = a, y(x_1) = b,$$

The coordinate function/or the admissible funciton $\phi_i(x), i = 1, 2, \ldots$ defined on $[x_0, x_1]$ must be

(a) linearly independent

(b) continuous

(c) smooth

(d) linearly independent, smooth and the functional be considered not along admissible curves $y = y(x)$ but only along all possible linear combinations of admissible functions

100. The integral equation, involving a parameter λ, $\phi(x) = \cos zx + \lambda \int_0^{\pi} \cos(x + \zeta) d\zeta$ has

(a) a unique solution if $\lambda = 1$, and an infinite number of solution if $\lambda = \frac{2}{\pi}$

(b) a unique solution if $\lambda = -1$, and an infinite number of solution if $\lambda = -\frac{2}{\pi}$

(c) a unique solution if $\lambda \neq -\frac{2}{\pi}$

(d) no solution if $\lambda = \pm\frac{2}{\pi}$

101. Consider the force free motion of a rigid body about a fixed point 0. Suppose $3A$, $5A$ and $6A$ are the principal moments of inertia at 0, and initially the angular velocity has components $\omega_1 = \sqrt{5},\ \omega_2 = 0,\ \omega_3 = \sqrt{5}$ about the corresponding principal axes; if the body ultimately rotates about the mean axis, then

(a) $\omega_1^2 + \omega_2^2 = 5$ (b) $5\omega_2^2 + g\omega_1^2 = 45$

(c) $\omega_3^2 = \omega_1^2$ (d) $\omega_2^2 \neq \omega_1^2$

102. Using Euler's dynamical equation for force-free motion of a rigid body, symmetrical about the Z-principal axis, with angular velocity $\bar{\omega} = (\omega_1, \omega_2, \omega_3)$, where $\omega_i, i = 1, 2, 3$, are the components along the three principal axes, it follows that

(a) ω_i = constant

(b) $\omega_2 = a \sin(\lambda t + b)$ with a, λ and b as constant

(c) ω_3 = constant

(d) $\omega_1^2 + \omega_2^2$ constant

UNIT - IV

103. Which of the following is/are cumulative distribution function(s) (c.d.f.) of random variable(s)?

(a) $F_1(x) = \begin{cases} 0, & x \leq 0 \\ e^{-x}, & x > 0 \end{cases}$

(b) $F_2(x) = \begin{cases} 0, & x \leq 0 \\ 1 - e^{-x}, & x > 0 \end{cases}$

(c) $F_3(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0 \end{cases}$

(d) $F_4(x) = \begin{cases} 0, & x < 0 \\ 1/2, & 0 \leq x < 1 \\ 1, & x \geq 0 \end{cases}$

104. Let X be a random variable taking values in a set E. Let $P(X > a + b \mid X > a) = P(X > b)$ for all $a, b \in E$. Then which of the following is a possible distribution of X?

(a) Poisson (b) Geometric

(c) Log - normal (d) Exponential

105. Let $\{X_n\}$ be a stationary Markov chain such that
$P(X_{i+1}=1|X_i=1)=p_1=1-P(X_{i+1}=0|X_i=1)$,
$P(X_{i+1}=1|X_i=0)=p_0=1-P(X_{i+1}=0|X_i=0)$,
and $P(X_1=1)=\pi_1=1-P(X_1=0)$. Then

(a) $\pi_1=p_1$ (b) $\pi_1=p_0$

(c) $\pi_1=\dfrac{p_0}{1-p_1+p_0}$ (d) $\pi_1=\dfrac{1}{2}$

106. Suppose X and Y are independent $N(0, 1)$ random variables. Let $U=\dfrac{X}{Y}$ and $V=\dfrac{X}{|Y|}$. Then

(a) U and V are independent
(b) U and V have the same distribution
(c) $P(U=V)=1/2$
(d) $P(U<V)=1/2$

107. Suppose $X_1, X_2, \ldots$ is a sequence of i.i.d. random variables where
$P(X_i=1)=p=1-P(X_i=0)$, $i=1, 2\ldots$
Let $Z=\dfrac{1}{500}\sum_{i=1}^{500}X_i$ and $\alpha=P(|Z-p|>0.1)$.
Then for all p

(a) $\alpha\le 0.1$ (b) $\alpha\le 0.05$

(c) $\alpha>0.01$ (d) $\alpha=0$

108. Suppose $X_1\sim U(0, \theta)$, $X_2\sim U(0, 1+\theta)$ and X_1 and X_2 are independent. Then

(a) min $\{X_1, X_2\}$ is sufficient for θ
(b) max $\{X_1, X_2\}$ is sufficient for θ
(c) max $\{X_1, X_2-1\}$ is sufficient for θ
(d) max $\{X_1+1, X_2\}$ is sufficient for θ

109. Suppose that we have $n\ge 1$ i.i.d. observations $X_1, X_2, \ldots, X_n$ each with a common $N(\mu, 1)$ distribution where $\mu\ge 0$ is unknown parameter. Then

(a) the maximum likelihood estimate and the uniformly minimum variance unbiased estimate for μ are he same
(b) the minimum variance unbiased estimate for μ is a consistent estimate.
(c) for any unbiased estimate for μ, there is another estimate for μ with a smaller mean squared error
(d) the maximum likelihood estimate for μ has smaller mean squared error than the estimate obtained by the method of moments.

110. Let $X_1, X_2, \ldots$ be i.i.d. observations from $N(\mu, \sigma^2)$ distribution with $-\infty<\mu<+\infty$ and $0<\sigma^2<\infty$ as unknown parameters. Then

(a) sample mean is an unbiased estimate for μ but sample median is not an unbiased estimate for μ.
(b) both sample mean and sample median are unbiased estimates for μ.
(c) sample mean has smaller variance than sample median.
(d) sample mean has smaller mean squared error than sample median.

111. Suppose $X\sim N(0, \sigma^2)$, Y has the exponential distribution with mean $2\sigma^2$ and X and Y are independent. We want to test at level α $H_0:\sigma^2\le 1$ versus $H_1:\sigma^2>1$. Then

(a) UMP test does not exist
(b) UMP test rejects H_0 when X^2+Y is large
(c) UMP test is a chi-square test
(d) UMP test is a t-test

112. Suppose that the probability distribution of a dis crete random variable X under two possible parameter values is as follows.

Parameter	1	2	3	4
θ_1	0.01	0.04	0.05	0.90
θ_2	0.80	0.10	0.05	0.05

Test $H_0: \theta=\theta_1$ versus $H_1: \theta=\theta_2$ at level $\alpha=0.05$. Then the most powerful test

(a) rejects H_0 if $x=1$ or $x=2$
(b) rejects H_0 if $x=3$
(c) has power larger than 0.85
(d) has power 0.5

113. In a Bayesian estimation problem of the Poisson mean λ, a gamma prior (with density proportional to $e^{-\beta\lambda}\ \lambda^{\alpha-1}$) is formulated. There is a sample of size n from the Poisson and the sample mean is $\bar{x}$. The posterior distribution of λ is
(a) a gamma distribution
(b) a poisson distribution
(c) has mean = $\frac{n\bar{x}+\alpha}{n+\beta}$
(d) has mean = $(n\bar{x}+\alpha)(n+\beta)$

114. Random variables X_1, X_2, X_3 are such that correlation (X_1, X_2) = correlation (X_2, X_3) = correlation $(X_3, X_1) = \rho$.
(a) ρ cannot be negative
(b) ρ can take any value between -1and +1
(c) $\rho \geq$-0.5
(d) ρ is either +1 or -1

115. Consider a linear model with four observations X_1, X_2, X_3, X_4 such that $E(X_1) = A+B+C$, $E(X_2) = A$, $E(X_3) = B$, $E(X_4) = A - B - C$ [where A, B, C, D are parameters]. Then
(a) $B + C$ is not estimable
(b) A,B,C are all estimable
(c) $A + B + C$ is estimable
(d) X_2 is the Best Linear unbiased estimate of A

116. In a survey of a population of $N = nk$ units, a sample of n units is to be drawn by systematic sampling with a random start between 1 and k and selecting every k^{th} unit. Then
(a) the sample mean is an unbiased estimate of the population mean.
(b) the variance of the sample mean cannot be estimated under this design
(c) if the N population units have been arranged at random, then the sample is equivalent to a simple random sample with replacement
(d) if the N population units have been arranged at random, then the sample is equivalent to a simple random sample without replacement.

117. Let D be a balanced incomplete block design with usual parameters v, b, r, k, λ . Which of the following statemetns is true?
(a) D is connected if $k \geq 2$
(b) The variance of the best linear unbiased estimator of an elementary treatment contrast under D is proportional to $2/r$
(c) The covariance between the best linear unbiased estimators of a pair of orthogonal treatment contrasts under D is zero.
(d) The efficiency factor of D relative to a randomized (complete) block design with replication r is strictly smaller than unity.

118. Suppose that we have a data set consisting of 25 observations, where each value is either 5 or 10.
(a) The mean of the data cannot be larger than the median.
(b) The mean of the data cannot be smaller than the median.
(c) The mean and the median for the data will be the same only, if the variance of the data is zero
(d) The mean and the median for the data will be different only if the range is 5.

119. Suppose that the LP problem maximise $c^T x$ subject to $Ax \leq b,\ x \geq 0$ admits a feasible solution and the dual minimise $b^T y$ subject to $A^T y \geq c$ $y \geq 0$ admits a feasible solution y_0. Then
(a) the dual admits an optimal solution
(b) any feasible solution x_0 of the primal and y_0 of the dual satisfies $b^T y_0 \leq c^T x_0$.
(c) the dual problem is unbounded.
(d) the primal problem admits an optimal solution

120. Let $X(t)$ be the number of customers in an $M/M/1$ queuing system with arrival rate $\lambda > 0$ and service rate $\mu > 0$. It is known that $\lim_{t\to\infty} P(X(t)-1) = \frac{1}{4}$. Which of the following is true?

(a) $\lim_{t\to\infty} E(X(t)=1) = \frac{1}{3}$

(b) $\lim_{t\to\infty} E(X(t)=1) = \frac{\lambda}{\mu}$

(c) $\lim_{t\to\infty} Var(X(t)=1) = \frac{1}{9}$

(d) $\lim_{t\to\infty} Var(X(t)=1) = \left(\frac{\lambda}{\mu}\right)^2$

ANSWERS

21.(c)	**22.**(a)	**23.**(b)	**24.**(a)	**25.**(a)	**26.**(a)	**27.**(d)
28.(c)	**29.**(b)	**30.**(d)	**31.**(a)	**32.**(a)	**33.**(a)	**34.**(a)
35.(c)	**36.**(c)	**37.**(d)	**38.**(c)	**39.**(d)	**40.**(d)	**41.**(d)
42.(d)	**43.**(a)	**44.**(b)	**45.**(a)	**46.**(c)	**47.**(c)	**48.**(d)
49.(d)	**50.**(c)	**51.**(a)	**52.**(d)	**53.**(d)	**54.**(d)	**55.**(c)
56.(d)	**57.**(d)	**58.**(c)	**59.**(b)	**60.**(b)	**61.**(a,c)	**62.**(a,c)
63.(b,d)	**64.**(b,d)	**65.**(a,b)	**66.**(a,d)	**67.**(d)	**68.**(a,b,d)	**69.**(b,d)
70.(b,d)	**71.**(a,b,c)	**72.**(a)	**73.**(a,,b,c)	**74.**(a,b)	**75.**(a,b,c)	**76.**(c,d)
77.(a,c)	**78.**(c,d)	**79.**(a,b,c)	**80.**(a,b)	**81.**(a,b,c)	**82.**(a)	**83.**(b,d)
84.(b,d)	**85.**(c)	**86.**(b,c)	**87.**(a,c)	**88.**(a,c,d)	**89.**(b, d)	**90.**(a,d)
91.(a,b,c,d)	**92.**(b,d)	**93.**(a,b,c,d)	**94.**(a,b)	**95.**(a)	**96.**(b,d)	**97.**(b,c)
98.(b,c)	**99.**(a,b,c,d)	**100.** (a,b)	**101.** (a,c,d)	**102.** (b, c, d)	**103.** (a)	**104.** (a,b)
105. (a,b,c,d)	**106.** (b,c)	**107.** (a,b)	**108.** (c)	**109.** (c,d)	**110.** (b,c,d)	**111.** (b,c)
112. (a,c)	**113.** (a,c)	**114.** (a, c)	**115.** (b, c)	**116.** (a,d)	**117.** (a,c)	**118.** (c,d)
119. (a, d)	**120.** (b,d)					

EXPLANATIONS

21. $\sum_j a_{ij} = 1, \forall i$ and $\sum_i a_{ij} = 1, \forall j\}$. Hence,total number of elements in S is

$$S = 5\times 4\times 3\times 2\times 1 = 5!$$

22. Total number of 4 digit numbers with no two digits common is $= 9\times 9\times 8\times 7 = 4536$

23. $det(A) = 1 \Rightarrow Rank(D) = n \geq 2 \neq 1$

24. $$f_n(x) = x^{1/n}, x \in [0,1]$$

$$\lim_{n\to\infty} f_n(x) = f(x) = 0 \quad if\ x = 0$$

$$= 1 \quad if\ 0 < x \leq 1$$

$\lim_{n\to\infty} f_n(x)$ exists for all $x \in [0,1]$

25. $A = (0,1)$ and $B = (1,8)$

$f : A \to B$ is given by $f(x) = 7x + 1$ is one-one and onto function from A to B.

26. w=(1,-1,1,1,-1)

$$\therefore\ wAw^T = (1,-1,1,1,-1)_{1\times 5} \times \begin{pmatrix} \xi^{-2} & 0 & 0 & 0 & 0 \\ 0 & \xi^{-1} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \xi & 0 \\ 0 & 0 & 0 & 0 & \xi^2 \end{pmatrix}_{5\times 5} \begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \\ -1 \end{pmatrix}_{5\times 1} = (\xi^{-2}, -\xi^{-1}, 1, \xi, -\xi^2)_{1\times 5} \begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \\ -1 \end{pmatrix}_{5\times 1}$$

$$= 1 + \xi + \xi^{-1} + \xi^2 + \xi^{-2} = 0$$

27. $\phi(1000) = 400$

28. $2^{M(4)+1} = 2,\quad 2^{M(4)+2} = 4,\quad 2^{M(4)+3} = 8,\quad 2^{M(4)} = 6$

As $100 = M(4)$. So answer is 6.

29. $(1+2+...+n)-1$ will be the dimension.(i.e) $= \dfrac{n(n+1)}{2} - 1 = \dfrac{n^2+n}{2} - 1$

30. $$\phi(x) = \begin{cases} 1-x & if\ x \leq 1 \\ x-1 & if\ 1 \leq x \leq 3/2 \\ 2-x & if\ 3/2 \leq x \leq 2 \\ x-2 & if\ x \geq 2 \end{cases}$$

is continuous on $\mathbb{R}$ but not differentiable only at $x = 1, \dfrac{3}{2}$ and 2.

31. $\lim_{n\to\infty}\frac{1}{n}\sin\frac{1}{n}=\lim_{n\to\infty}\frac{\sin\frac{1}{n}}{n}=0$, Hence we have only one limit point 1.

32. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}=\log 2$

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{n(n+1)}=-\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}-\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}-...=-\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)-\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)-...$$

$$=-1-2\left(-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-...\right)=-1-2(\log 2-1)$$

$$=-1-2\log 2+2=1-2\log 2$$

33. Given that, $v(x,y)=3xy^2$

$\therefore\ \frac{\partial v}{\partial x}=3y^2\Rightarrow\frac{\partial^2 v}{\partial x^2}=0$, and $\frac{\partial v}{\partial y}=6xy\Rightarrow\frac{\partial^2 v}{\partial y^2}=6x$, $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=0-6x=-6x\neq 0,\forall x\neq 0$.

Hence, $v(x,y)=3xy^2$ is not harmonic function for any choice of u.

34. $\det(V,W)=\begin{vmatrix}v_1 & v_2\\ w_1 & w_2\end{vmatrix}$, Derivative of $\det(V,W)=\begin{vmatrix}v'_1 & v'_2\\ w_1 & w_2\end{vmatrix}+\begin{vmatrix}v_1 & v_2\\ w'_1 & w'_2\end{vmatrix}=\det(H,W)+\det(V,K)$

35. $T:W\to W$ given by $(Tp)(x)=p'(x)$

where p' is the derivative of p and {1,x,x2,x3} is a basis of W,

then
$$T(1)=0=0\cdot 1+0\cdot x+0\cdot x^2+0\cdot x^3$$
$$T(x)=1=1\cdot 1+0\cdot x+0\cdot x^2+0\cdot x^3$$
$$T(x^2)=2x=0\cdot 1+2\cdot x+0\cdot x^2+0\cdot x^3$$
$$T(x^3)=3x^2=0\cdot 1+0\cdot x+3\cdot x^2+0\cdot x^3$$

The required maxtrix is $\begin{bmatrix}0&1&0&0\\0&0&2&0\\0&0&0&3\\0&0&0&0\end{bmatrix}_{4\times 4}$

36.
$$[Q(\sqrt{2}+\sqrt[3]{2}):Q]=[Q(\sqrt{2}+\sqrt[3]{2}):Q(\sqrt{2})][Q\sqrt{2}:Q]$$
$$6=[Q(\sqrt{2}+\sqrt[3]{2}):Q(\sqrt{2})]\times 2$$
$$[Q(\sqrt{2}+\sqrt[3]{2}):Q(\sqrt{2})]=\frac{6}{2}=3$$

37. $\sum_{n=0}^{\infty}\frac{z^{2n}}{2^n}$ will converges if $\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right|<1 \Rightarrow Lt\left|\frac{2^{-(n+1)}z^{2(n+1)}}{2^{-n}z^{2n}}\right|<1 \Rightarrow |z|^2<2 \Rightarrow |z|<\sqrt{2}$

38. $J_n=\{(g_1,g_2,...,g_n):g_1,g_2,...,g_n=e\} \Rightarrow g_n=g_{n-1}^{-1}g_{n-2}^{-1}...g_1^{-1}$, So it is a normal subgroup.

39. $I_1=<x^4+3x^2+2>$, $I_2=<x^3+1>$

Here, $x^4+3x^2+2=(x^2+2)(x^2+1)$

x^4+3x^2+2 is reducible over Q. So, $F_1=\frac{Q[x]}{I_1}$ is not a field.

$x^3+1=(x+1)(x^2-x+1)$, x^3+1 is reducible over Q. I_2 is not maximal Ideal hence $F_2=\frac{Q[x]}{I_2}$ is not field.

40. $o(G)=77=11\times 7$. Let $p=7, q=11$

Then, $p \nmid q-1$. Hence, G is cyclic group. Hence, G is abelian group. Therefore, $Z(G)=G$

Hence, $\mathbb{Z}_{77}\cong Z(G)$

41. $u'(t)=P(u(t)):u(0)=1$, As polynomial is contained in the region and initial value is given at 0, So in any interval containing 0. IVP will have a unique solution.

42. $1-2t=0$ at $t=0$; $\sin 2\pi t=0$ at $t=0$, $\cos 2\pi t=0$ at $t=1/4$ but $t=1$ is never 0 in the interval.

43. $(x^2+2y)u_{xx}+(y^3-y+x)u_{yy}+x^2(y-1)u_{xy}-3u_x+u=0$

Comparing with $Rr+Ss+Tt+f(x,y,z,p,q)=0$

$$R=(x^2+2y),\quad S=x^2(y-1),\quad T=(y^3-y+x)$$

$$S^2-4RT=[x^2(y-1)]^2-4(x^2+2y)(y^3-y+x)$$

Putting $x=y=1$,

we get $S^2-4RT=0-4\times(3)\times(1)=-12<0$

Hence it is elliptical. It has two characteristic curve at $x=1, y=1$.

44. $4u_{xx}-u_{yy}=0 \Rightarrow 4D^2-D'^2=0$

$\therefore$ Auxiliarly equation is $4m^2-1=0 \Rightarrow m=\pm\frac{1}{2}$

$$u(x,y)=f\left(y+\frac{1}{2}x\right)+g\left(y-\frac{1}{2}x\right)=f(x+2y)+g(x-2y)$$

45. $f(x)=e^{-x}$, $f'(x)=-e^{-x}$, $f''(x)=e^{-x}$, and $f'''(x)=-e^{-x}$

By Taylor's approximation $g(x)=f(0)+(x-0)f'(0)+\frac{(x-0)^2}{2!}f''(0)+\frac{(x-0)^3}{3!}f'''(0)$

$$g(x)=1-x+\frac{x^2}{2}-\frac{x^3}{6}$$

$\therefore$ $g\left(\frac{1}{3}\right)=1-\frac{1}{3}+\frac{1}{18}-\frac{1}{162}=\frac{53}{81}$ which is greater than 0 but less than 1.

46. Here, $f(x,y,y') = y'^2 - y^2$

$$\frac{\partial F}{\partial y} = -2y, \frac{\partial F}{\partial y'} = 2y'$$

Euler's Lagrange's equation is $\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$

$$y'' + y = 0$$

A.E is $\lambda^2 + 1 = 0 \Rightarrow \lambda = \pm i$

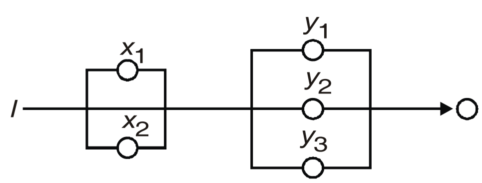
$\therefore$ Complementary function is $y(x) = c_1 \cos x + c_2 \sin x$,where c_1 and c_2 are arbitrary function.

Now,applying boundary conditions,we get $1 = c_1 \Rightarrow c_2$ is arbitrary. $\therefore$ y has infinitely many solutions.

47. Resolvent kernel of $\phi(x) = x + 2\int_0^x e^{x-\xi}\phi(\xi)d\xi$ is $R = (x,\xi,\lambda) = e^{(1+2)(x-\xi)} = e^{3(x-\xi)}$

48. Degree of freedom= No. of independent tuples required to reresent the particles in the space. Due to 3 rigid massless rods degree of freedom is decreased by 3 and a point in space is represented by 3 tuples. So, degree of freedom $= 3 \times 3 - 3 = 6$

49.



According to the figure,system will work if at least one unit of A and atleast one unit of B will work simultaneously.

$$P(\text{system is in working}) = \{1 - P(A \text{ failed})\} \times \{1 - P(B \text{ failed})\}$$

$$= \{1 - P(\bar{x}_1\bar{x}_2)\}\{1 - P(\bar{y}_1\bar{y}_2\bar{y}_3)\} = \left(1 - \frac{1}{2} \times \frac{1}{2}\right)\left(1 - \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) = \frac{21}{32}$$

50. $T_n = \sqrt{n}(Y_n - Z_n)$,

then $E(T_n) = 0$

$$V(T_n) = V\left[\frac{1}{\sqrt{n}}\sum_{i=1}^{n}(x_{2i-1} - x_{2i})\right] = \frac{1}{n}E\left[\sum_{i=1}^{n}(x_{2i-1} - x_{2i})\right]^2$$

$$= \frac{1}{n}\left[\sum_{i=1}^{n}E(x_{2i-1} - x_{2i})^2\right] (\because x_1, x_2, \ldots \text{are i.i.d.})$$

$$= \frac{1}{n}n[E(X_{2i-1}^2) + E(X_{2i}^2)] = 2\sigma^2$$

Therefore,by central limit theorem, $T_n \sim N(0, 2\sigma^2)$

53. $ML \in \hat{N}$, consider the ratio

$$R(N) = \frac{P_N(x)}{P_{N-1}(x)} = \frac{N-n}{N} \cdot \frac{N-M}{N-M-n+x}$$

$P_N(x)$ reaches its maximum value where $N \simeq \dfrac{nM}{x}$

$$\hat{N} = \frac{30 \times 40}{10} = 120$$

55. For independent normal random variables.

$$\pi_c = P\{(x_i - x_j)(y_i - y_j) > 0\} = \pi_d = P\{(x_i - x_j)(y_i - y_j) < 0\}$$

i.e., $\gamma = \pi_c - \pi_d = 0$

Thereforce, $E(\gamma) = 0$

57. P (lot will not be passed) = p (chosen box has defective batteries) $\times P$ (one or two defective batteries will be selected)

$$= \frac{1}{10} \times \left[\frac{{}^2C_1 \times {}^8C_1}{{}^{10}C_2} + \frac{{}^2C_2}{{}^{10}C_2} \right] = \frac{1}{10} \times \frac{17}{45} = \frac{17}{450}$$

59. Maximum $Z = x_1 + x_2$

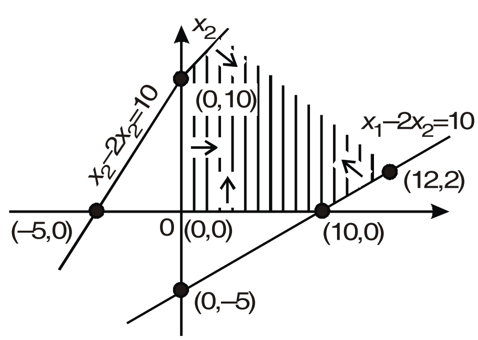
Subject to

$x_1 - 2x_2 \leq 10$

$x_2 - 2x_1 \leq 10$

$x_1, x_2 \geq 0$

Then the graph is given below



Now,

$(Z)_{(0,0)} = 0 + 0 = 0$

$(Z)_{(10,0)} = 10 + 0 = 10$

$(Z)_{(0,10)} = 0 + 10 = 10$

$(Z)_{(12,2)} = 12 + 2 = 14$

Hence,the LP problem has unbounded solution.

60. $\lambda = 3$ and $\mu = 6$

$\therefore$ $$\rho = \frac{\lambda}{\mu} = \frac{3}{6} = \frac{1}{2}$$

$$\therefore \qquad p_0 = 1-\rho = 1-\frac{1}{2} = \frac{1}{2}$$

$$p_1 = \rho p_0 = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$$

$$p_2 = \rho p_1 = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8}$$

$$p_3 = \rho p_2 = \frac{1}{2}\cdot\frac{1}{8} = \frac{1}{16}$$

$$p_4 = \frac{1}{2}\cdot\frac{1}{16} = \frac{1}{32}$$

$$\therefore \qquad \lim_{t\to\infty} P(x(t)\ge 5) = 1 - p_0 - p_1 - p_2 - p_3 - p_4$$

$$= 1-\left(\frac{1}{2}\right)-\left(\frac{1}{4}\right)-\left(\frac{1}{8}\right)-\left(\frac{1}{16}\right)-\left(\frac{1}{32}\right) = 1-\left(\frac{31}{32}\right) = \frac{1}{32}$$

61. $f(x) = |\cos x| + |\sin(x-2)|$

But $|\cos x|$ is not differentiable only at $x = (2n+1)\frac{\pi}{2}, n\in\mathbb{Z}$ and $|\sin(x-2)|$ is not differentiable only at

$$x-2 = n\pi, n\in\mathbb{Z}, \qquad x = n\pi+2, n\in\mathbb{Z}$$

62. (a) $X_1 = \{(x,y): |x|\le 5, |y|\le 10\}$

Let $\quad (x_1,y_1),(x_2,y_2)\in X_1$

$\Rightarrow \quad |x_1|\le 5, |x_2|\le 5$

and $\quad |y_1|\le 10, |y_2|\le 10$

Let $t>0$, consider

$$t(x_1,y_1)+(1-t)(x_2,y_2) = (tx_1,ty_1)+((1-t)x_2,(1-t)y_2) = (tx_1+(1-t)x_2, ty_1+(1-t)y_2)$$

$$|tx_1+(1-t)x_2| \le |tx_1| + |(1-t)x_2| = t|x_1| + (1-t)|x_2| \le t\cdot 5 + (1-t)\cdot 5 = 5t+5-5t = 5$$

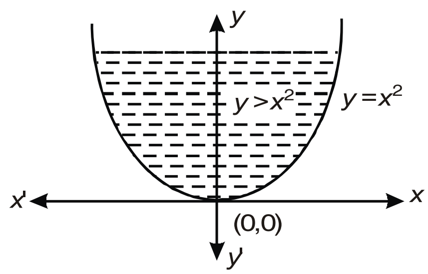
$$\therefore \qquad |tx_1+(1-t)x_2| \le 5$$

$$|ty_1+(1-t)y_2| \le |ty_1| + |(1-t)y_2| = t|y_1| + (1-t)|y_2| \le t\cdot 10 + (1-t)\cdot 10 = 10t+10-10t = 10$$

$$\therefore \qquad |ty_1+(1-t)y_2| \le 10$$

Hence, $t(x_1,y_1)+(1-t)(x_2,y_2)\in X_1$ for $t>0$; X_1 is convex set.

(c) $$X_3 = \{(x,y): y\ge x^2\}$$



The line joining any two points of X_3 is always lies in X_3 Hence, X_3 is a convex set.

63. $|x-y| = 0 \Rightarrow x = y$

$|x^3 - y^3| = 0 \Rightarrow x = y$ on R but not in C(set of complex numbers)

64. $Y = \prod_{j=1}^{\infty} X_j = 2\times 2\times 2\times \ldots N \quad times = 2^N$ which is uncountable. Union of countable set is countable.

65. (a) $\lim\limits_{n\to\infty} n\log\left(1+\frac{1}{n+1}\right) = \lim\limits_{n\to\infty}\log\left(1+\frac{1}{n+1}\right)^n = \log\lim\limits_{n\to\infty}\left(1+\frac{1}{n+1}\right)^n = \log e^{\lim\limits_{n\to\infty}\left(1+\frac{1}{n+1}-1\right)^n}$

$$= \log e^{\lim\limits_{n\to\infty}\left(\frac{1}{1+n}\right)^n} = \log e = 1$$

(b) $\lim\limits_{n\to\infty}(n+1)\log\left(1+\frac{1}{n}\right) = \lim\limits_{n\to\infty}\log\left(1+\frac{1}{n}\right)^{n+1} = \log\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+1}$

$$= \log e^{\lim\limits_{n\to\infty}\left(1+\frac{1}{n}-1\right)^{(n+1)}} = \log e^{\lim\limits_{n\to\infty}\frac{n+1}{n}} = \log e = 1$$

66. They are inequalities of limsup and liminf.

67. By bounded convergence theorem, only statement 4 is valid.

68. $\|x\|_1 \geq \|x\|_2 \geq \|x\|_\infty$ **.** But $\|x\|_1 \leq d\|x\|_\infty$, also $\|x\|_1 \leq \sqrt{d}\,\|x\|_2$

69. $\underset{(x,y)\to(0,0)}{Lt} f(x,y) = \underset{(x,y)\to(0,0)}{Lt}(3x-2y+x^2, 4x+5y+y^2) = (0,0)$

$\Rightarrow f$ is continuous.

$$D_p(t) = \underset{t\to 0}{Lt}\frac{f(p+tu)-f(p)}{t}$$

$$= \underset{t\to 0}{Lt}\frac{(3t\cos\theta - 2t\sin\theta + t^2\cos^2\theta, 4t\cos\theta + 5t\sin\theta + 25t^2\sin^2\theta)}{t}$$

$$= (3\cos\theta - 2\sin\theta, 4\cos\theta + 5\sin\theta)$$

$f(x,y) = (3x - 2y + x^2, 4x + 5y + y^2)$

$\therefore$ $f_1(x,y) = 3x - 2y + x^2$

$f_2(x,y) = 4x + 5y + y^2$

$$\therefore \qquad Df(0,0) = \begin{vmatrix} (3+2x)_{(0,0)} & (-2)_{(0,0)} \\ 4_{(0,0)} & (5+2y)_{(0,0)} \end{vmatrix} = \begin{vmatrix} 3 & -2 \\ 4 & 5 \end{vmatrix} = 15+8 = 23 \neq 0$$

Hence, f is differentiable at (0,0) and $Df(0,0)$ is invertible.

70. (x, y) has all member of $Q\times Q$ and some members of $(R-Q)\times(R-Q)$. So (b) is dense in R^2.

$xy \neq 0$. All points except x and y axis are in it. It is dense in R^2.

71. Continuity and uniform continuity of every f is guaranteed. As all differentiable functions are also continuous, so every differentiable is contained in it.

72. (c,d)

$$A = \begin{bmatrix} a_1^2 & a_1a_2 & \cdots & a_1a_n \\ a_2a_1 & a_1^2 & \cdots & a_2a_n \\ \vdots & \vdots & \vdots & \vdots \\ a_na_1 & a_na_2 & \cdots & a_n^2 \end{bmatrix}$$

Rank of symmetric matrix is 1. So one eigen value is non zero and remaining zero. As trace of A is $a_1^2 + a_2^2 + a_3^2 + \cdots + a_n^2$. So, 1 eigenvalue is $a_1^2 + a_2^2 + a_3^2 + \cdots + a_n^2$ and $n-1$ are zero. So, matrix A is positive semidefinite if $n > 1$, but for $n = 1$, it is positive definite. $\Rightarrow$ A is singular.

73. Let $\lambda_1, \lambda_2, ..., \lambda_n$ are eigen values of matrix A and $\mu_1, \mu_2, ..., \mu_n$ are eigen values of matrix B.

Since, A and B are positive definite matrix. Hence, $\lambda_i > 0, \quad \forall i \;\; \mu_j > 0, \quad \forall j$

eigen values of $A + B$ are $\lambda_1 + \mu_1,\; \lambda_2 + \mu_2, ..., \lambda_n + \mu_n$. Also $\lambda_i + \mu_i > 0,\; \forall i.$, Hence, $A + B$ is positive definite matrix. Also, eigen vlaues of $A^2 + I$ are $\lambda_i^2 + I$ which are always positive. Hence, $A^2 + I$ is positive definite.

74. $T^2 = \lambda T \Rightarrow T(X) = \lambda X \Rightarrow T \circ T(X) = T(\lambda X) = \lambda^2 X = \lambda T(X)$

$\|T(X)\| = |\lambda|\|X\|, \quad \forall x \in R^n \quad \Rightarrow \|T(X)\| = \|X\| \Rightarrow |\lambda| = 1 \Rightarrow \lambda = \pm 1$

$T^2 = \lambda T \Rightarrow |T|^2 = \lambda^n |T| \Rightarrow |T| = 0, \lambda^n$

$\|T(X)\| \geq \|X\| \Rightarrow |\lambda| > 1$. So transformatiion is non singular.

75. $X, Y \in M \Rightarrow XA = AX \quad and \; YA = AY$

$\forall a \in R \quad (aX+Y)A = aXA + YA = A(aX+Y)$ is a subspace.

$(aX+Y) + A = A + (aX+Y)$ is a subspace.

$trace(A(aX+Y)) = atrace(AX) + trace(AY) = a.0 + 0 = 0$ is a subspace.

$\det A(aX+Y) \neq 0$ even if $\det AX = 0$ *and* $\det AY = 0$ is not a subspace.

76. $$|B - \lambda I| = \begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 1 & 0 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow -\lambda(\lambda^2)+1=0 \Rightarrow \lambda^3-1=0$. So, For polynomial of degree upto 2, we can have freedom as $A^3 = I$.

So, olynomial can have upto 2^{nd} power (i.e) 0,1 or 2. So, dimenion of vector space is $d = 3$.

77. (a,c) N is a nilpotent matrix of index 3. Hence,all the eigen values of N are 0 and it is not similar to a diagonal matrix.

79. (a,b,c) $f\left(\frac{1}{n}\right) = \frac{2n}{3n+1} \Rightarrow f(z) = \frac{2\cdot\frac{1}{z}}{3\cdot\frac{1}{z}+1} = \frac{2}{3+z} \Rightarrow f(0) = \frac{2}{3}$.

$f(z)$ has simple pole at $z=-3$, $\quad f(3) = \frac{2}{3+3} = \frac{2}{6} = \frac{1}{3}$

80. (a,b) Re($f(z)$) is bounded and $f(z)$ is entire., so $f(z)$ will be constant. $\Rightarrow im\ f$ is constant. $\Rightarrow f$ is constant.

81. Consider the analytic function $f: D \to D$ defined by

Then,
$$f(0) = \frac{1}{2} \text{ and } f\left(\frac{1}{2}\right) = 0$$
$$f'(z) = -1$$
$$\therefore \quad \left|f'\left(\frac{1}{2}\right)\right| = |-1| = 1 < \frac{4}{3}$$

82. $f(z) = \frac{z}{3z+1} = \frac{x+iy}{(3x+1)+i3y} = \frac{(x+iy)((3x+1)+i3y)}{(3x+1)+i3y} = \frac{3(x^2+y^2)+x}{(3x+1)^2+(3y)^2} + i\frac{y}{(3x+1)^2+(3y)^2}$

Sign of imaginary part remains unchanged after the transformation. So, $f(z)$ maps H^+ onto H^+ and H^- onto H^-.

83. $f(z) = \frac{e^z+1}{e^z-1} = \frac{\left(2+z+\frac{z^2}{2!}+...\right)}{\left(z+\frac{z^2}{2!}+...\right)} = \frac{2+z+z^2/2!+...}{z(1+z/2!+...)}$

$\Rightarrow \lim_{z\to 0} zf(z) = 2 \neq 0 \Rightarrow z = 0$ is a simle pole. Also, $f(z) = \frac{1}{z}\left[2+z+\frac{z^2}{2!}+...\right]\left[1-\left(\frac{z}{2!}+...\right)+...\right]$

coefficient of $1/z$ in $f(z) = 2$ so residue =2.

84. $O(H) = 2, O(K) = 4 \quad i_H(K) = 2 \Rightarrow H$ is normal in K.

85. $f(x) = (x^2+1)(x+1), \quad g(x) = (x^2+1)(x-1) \Rightarrow < f(x)+g(x) > = < x^2+1 >$

86. $I_1 =< x^2 + 1 > \; x^3 - x^2 + x - 1 = (x^2+1)(x-1)$, is reducible.

$R_1 = Q[x]/I_1$ is a field. $R_2 = Q[x]/I_2$ is not field. The ideal $< x^2 + 1 >$ is maximal over $Q[x]$, so it is an integral domain.

87. We have $$G = \mathbb{Z}_{10} \times \mathbb{Z}_{15}.$$

The number of elements of order 2 = $\phi(2)\times\phi(1) = 1\times 1 = 1$

The number of elements of order 5 $= \phi(1)\times\phi(5)+\phi(5)\times\phi(1)+\phi(5)\times\phi(5) = 1\times 4+4\times 1+4\times 4 = 24$

88. $\|f\|_\infty = \sup\{|f(x)| : x \in [0,1]\}$ shows completeness property, and so does options in (c) and (d).

89. (c,d) The open set $(-2,0)$ belongs to $(-2,0]$. So in subspace topology $(-2,0]$ is an open set. (-2, 0) is an open set.

90. (b,d) $A_1 = \{(x,y) : x^2 + y^2 < 1\}$, $A_2 = \{(1,y) : y \in \mathbb{R}\}$, $A_3 = \{(0,2)\}$

and $$f(x) = a_j \text{ for } x \in A_j, j = 1,2,3,...$$

Now, $(1,0) \in A_1$ and $(1,0) \in A_2$ $\therefore$ $f(1,0) = a_1$ and $f(1,0) = a_2$

but f is continuous. Hence, $a_1 = a_2$ and a_3 is arbitrary.

91. Given boundary value problem is $y'' + \lambda y = 0,\; y(0) = 0 = y(1)$

(a) green function $G(x,\xi), 0 \le x, \xi \le 1$ is always symmetric in x and ξ.

(b) green function $G(x,\xi), 0 \le x, \xi \le 1$ is continuous at $x = \xi$.

(c) $$\left.\frac{\partial G(x,\xi)}{\partial x}\right|_{x=\xi^-} - \left.\frac{\partial G(x,\xi)}{\partial x}\right|_{x=\xi^+} = -\frac{1}{p_0(x)} = -\frac{1}{1} = -1 \quad [\because p_0(x) = 1]$$

93. Given that $y_1(x)$ and $y_2(x)$ form a fundamental set of solutions to the differential equation

$$\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + q(x)y = 0, a \le x \le b,$$

where $p(x)$ and $q(x)$ are continuous in $[a,t]$ and $x_0 \in (a,b)$

(a) Let $y_1(x)$ and $y_2(x)$ have a local maximum at x_0, then

$$y_1'(x_0) = y_2'(x_0) = 0$$

$$\therefore \qquad W(y_1(x_0), y_2(x_0)) = \begin{vmatrix} y_1(x_0) & y_2(x_0) \\ y_1'(x_0) & y_2'(x_0) \end{vmatrix} = \begin{vmatrix} y_1(x_0) & y_2(x_0) \\ 0 & 0 \end{vmatrix} = 0$$

Hence, $y_1(x)$ and $y_2(x)$ are linearly dependent but $y_1(x)$ and $y_2(x)$ are forms fundamental set of solutions. Which is a contradiction. Hence, both $y_1(x)$ and $y_2(x)$ cannot have local maxima at $x_0 \in (a,b)$.

(b) Let both $y_1(x)$ and $y_2(x)$ have local minima at $x_0 \in (a,b)$. Then, $y_1'(x_0) = y_2'(x_0) = 0$

$$W(y_1(x_0),y_2(x_0)) = \begin{vmatrix} y_1(x_0) & y_2(x_0) \\ y_1'(x_0) & y_2'(x_0) \end{vmatrix} = \begin{vmatrix} y_1(x_0) & y_2'(x_0) \\ 0 & 0 \end{vmatrix} = 0$$

Hence,again $y_1(x)$ and $y_2(x)$ are linearly dependent but $y_1(x)$ and $y_2(x)$ are forms fundamental set of solutions. Which is a contradiction. Hence,both $y_1(x)$ and $y_2(x)$ cannot have local minima at

(c) Let $y_1(x)$ have a local maximum at x_0 and $y_2(x)$ have local minima at x_0 simultaneously.

Then, $$y_1'(x_0) = y_2'(x_0) = 0$$

$\therefore$ $$W(y_1(x_0),y_2(x_0)) = \begin{vmatrix} y_1(x_0) & y_2(x_0) \\ y_1'(x_0) & y_2'(x_0) \end{vmatrix} = \begin{vmatrix} y_1(x_0) & y_2(x_0) \\ 0 & 0 \end{vmatrix} = 0$$

Hence, again $y_1(x)$ and $y_2(x)$ are linearly dependent but $y_1(x)$ and $y_2(x)$ are forms fundamental set of solutions. Which is a contradiction. Hence $y_1(x)$ cannot have local maximum at $x_0 \in (a,b)$. and $y_2(x)$ cannot have local minimum at $x_0 \in (a,b)$.

Hence,option (c) is correct.

(d) Let both $y_1(x)$ and $y_2(x)$ vanishes at x_0 simultaneously.

Then, $$y_1(x_0) = y_2(x_0) = 0$$

$\therefore$ $$W(y_1(x_0),y_2(x_0)) = \begin{vmatrix} y_1(x_0) & y_2(x_0) \\ y_1'(x_0) & y_2'(x_0) \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

Hence,again $y_1(x)$ and $y_2(x)$ are linearly dependent but $y_1(x)$ and $y_2(x)$ are forms fundamental set of solutions. Which is a contradiction.

94. $$uu_x + yu_y = x$$

$\therefore$ Lagrange's auxiliary equation is

$$\frac{dx}{u} = \frac{dy}{y} = \frac{du}{x}$$

$$\frac{dx}{u} = \frac{du}{x}$$

$\Rightarrow$ $$xdx = u\,du$$

On integration,we get $$x^2 - u^2 = c^2$$

Next, $\frac{dx}{u} = \frac{dy}{y}$

$$\frac{dx}{\sqrt{x^2 - c^2}} = \frac{dy}{y}$$

On integration,we get

$$\ln(x + \sqrt{x^2 - c^2}) = \ln y + \ln d$$

$$\ln\left(\frac{x+\sqrt{x^2-c^2)}}{y}\right)=\ln d$$

$$\frac{x+\sqrt{x^2-x^2+u^2)}}{y}=d$$

$$\frac{x+u}{y}=d$$

$$\frac{y}{x+u}=\frac{1}{d}=c_1$$

Hence,solution is

$$f\left(x^2-u^2,\frac{y}{x+u}\right)=0$$

where $f:\mathbb{R}^2\to\mathbb{R}$ is c^1 and $\nabla f\neq(0,0)$ at every point

or $$x^2-u^2=g\left(\frac{y}{x+u}\right), g\in c^1(\mathbb{R})$$

$\Rightarrow$ $$u^2=g\left(\frac{y}{x+u}\right)+x^2, g\in c^1(\mathbb{R})$$

99. Given functional is $I(y)=\int_{x_0}^{x_1}F\left(x,y,\frac{dy}{dx}\right)dx$ with $y(x_0)=a, y(x_1)=b$

(a) The coordinate function/or the admissible function $\phi_i(x)$; $i=1,2,...$ defined on $[x_0,x_1]$ must be linearly independent.

(b) The coordinate function/or the admissible function $\phi_i(x)$; $i=1,2,...$ defined on $[x_0,x_1]$ must be continuous.

(c) The coordinate function/or the admissible function $\phi_i(x)$; $i=1,2,...$ defined on $[x_0,x_1]$ must be smooth.

(d) The coordinate function/or the admissible function $\phi_i(x)$; $i=1,2,...$defined on $[x_0,x_1]$ must be linearly independent, smooth and the functional be considered not along admissible curves $y=y(x)$ but only along all possible linear combinations of admissible functions.

104. The two distributions geometric and exponential have the property of memorylessness

For exponential distribution, $f(x)=\frac{1}{\theta}e^{-x/\theta}, x>0, \theta>0.$

$$P(x>a+b\,|\,x>0)=\frac{P(x>a+b}{P(x>a)}=\frac{e^{(a+b)/\theta}}{e^{a/\theta}}=e^{b/\theta}$$

and for geometric distribution,

$$p(x) = pq^x, x = 1, 2, ...$$

$$P(x, a+b \mid x > a) = \frac{P(x > a+b)}{P(x > a)} = \frac{q^{a+b}}{q^a} = q^b$$

106. Both U and V have the Cauchy's distribution $C(1,0)$ and U and V are related by the equation

$$U = \begin{cases} +V, y > 0 \\ -V, y < 0 \end{cases}$$

Therefore, $$P(U = V) = \frac{1}{2}$$

108. For $U(0,\theta)$, max $\{x_1\}$ is sufficient which for $U(0,1+\theta)$, max $\{x_2 - 1\}$ is sufficient. Combinedly max $\{X_1, X_2 - 1\}$ is sufficient for θ.

109. Take the case of smaller number of observations (say 5 observations) and think on various situations. (5,5,5,5,5; M = Me, Var = 0), (5,5,5,5,10; M > Me)... (5,10,10,10,10; M < Me),(10,10,10,10,10; M = Me, Var = 0)

110. $$\text{MSE}(\hat{\theta}) = \text{Var}(\hat{\theta}) + [\text{bias}(\hat{\theta})]^2$$

and sample mean and median both are unbiased estimators of population mean in normal distribution.
Also, Var(Me) > Var(Mean)

$$\Rightarrow \quad \text{Ms} \in (\text{Me}) > \text{Ms} \in (\text{Mean})$$

111. $X^2 + Y$ follows χ^2 distribution which is UMP test and the critical region of this test is given by $c \geq \chi^2_{n-1,1-\frac{\alpha}{2}}$, where n is the sample size. Note that χ^2 is non-centralized in this case.

112. ln first case power at $\alpha = 0.05$

$$P = (X = 1,2/H_1) = 0.90$$

which is maximum and > 0.85

113. The likelihood function for λ is

$$L(x_1, x_2, ..., x_n; \lambda) = \prod_{i=1}^{n} \frac{\lambda^{xi} e^{-\lambda}}{x_i!} = \frac{\lambda^{n\bar{x}} e^{-n\lambda}}{\prod_{i=1}^{n} x_0!}$$

Therefore,the point density with prior distribution of λ i.e.,posterior distribution of λ is given by

$$f(\lambda) = k(\lambda^{n\bar{x}} e^{-n\lambda})(e^{-\beta\lambda}\lambda^{\alpha-1})$$

$$= k\lambda^{n\bar{x}+\alpha-1} e^{-\lambda(n+\beta)}$$

Which is Gamma distribution with mean $\frac{n\bar{x}+\alpha}{n+\beta}$

114. Suppose $P < 0$,then cov $(X_1, X_2) < 0$ and cov $(X_1, X_3) < 0$.
There is negative correlation of X_1 with X_2 and X_3 both
There is positive correlation between X_2 and X_3,which is not possible from the equation.

115. As $\hat{A} = X_2,\ \hat{B} = X_3,\ \hat{C} = X_1 - X_2 - X_3$ are unbiased estimates and also $(A + \hat{B} + C) = X_1$
But X_2 is not BLUE for A,BLUE of A will be a linear combination of X_1, X_2 and X_4.

117. For any BIBD,$k \geq 2$ is necessary condition for connectivity but not sufficient condition. Also,variance 1 BLUE of treatment contrast is not inversely proportional to r.

119. If LP problem Maximum $Z = C^T x$
Subject to $AX \leq b$
$X \geq 0$
Admits a feasible solution and the dual problem
Minimum $W = b^T y$
Subject to $A^T Y \geq c$
$Y \geq 0$
Admits a feasible solution y_0. Then primal and dual both admits an optimal solution.

120. $X(t)$ be the number of customer in an $M|M|1$ Queuing system with arrival rate $\lambda > 0$ and ser vice rate $\mu > 0$.

Also, $$\lim_{t\to\infty} P(X(t) = 1) = \frac{1}{4}$$

Then, $$\lim_{t\to\infty} E(X(t) = 1) = \frac{1}{3}$$

and $$\lim_{t\to\infty} Var(X(t) = 1) = \frac{1}{9}$$

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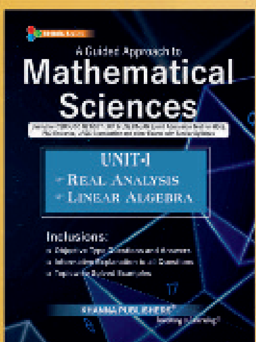

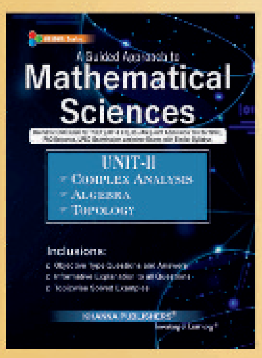

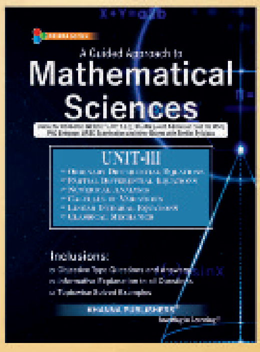

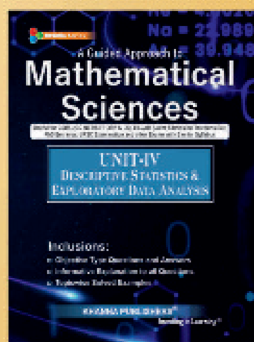

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